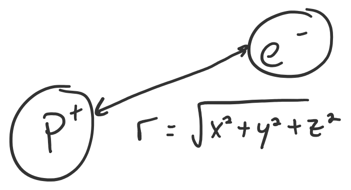
Lecture 10

Wednesday, September 13, 2023 10:02 AM

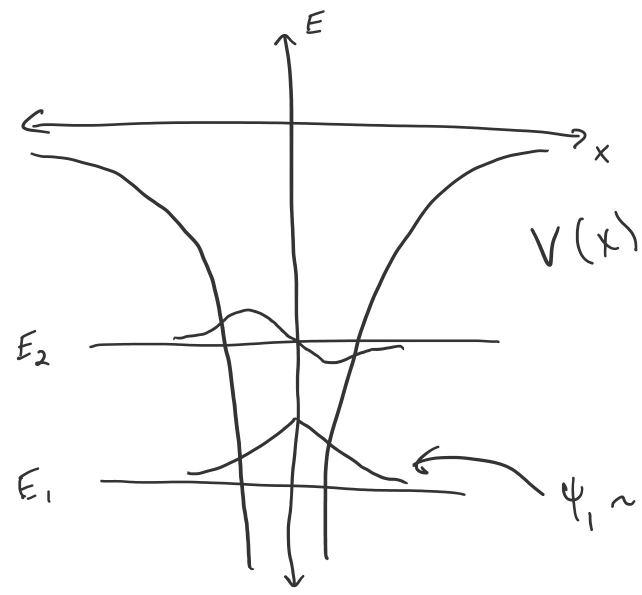
One More "Box" = the H-atom



$$V(r) = -\frac{e^2}{4\pi\varepsilon_0 r}$$

$\sqrt{x^2} = |x|$

Let's look at a 1-D cut along x-axis, although this box is 3-D (x, y, z)



$$V(x) = \frac{-e^2}{4\pi\varepsilon_0 |x|}$$

$\psi_1 \sim e^{-|x|/a_0}$

$n=2 \quad \psi = 2p_x$

$n=1 \quad \psi = 1s$

In 3-D, solve $\mathcal{H}\psi_n = E_n\psi_n$

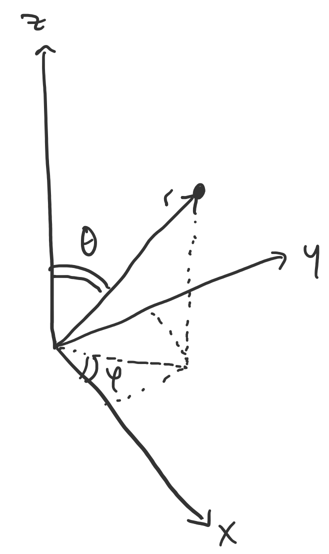
$$\left\{-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) - \frac{e^2}{4\pi\varepsilon_0 r}\right\}\psi = \overbrace{E_{n,\ell,m_\ell},}^{\text{3 quantum numbers for 3D space}}{}_{m_s} \cdot \psi$$

$r = \sqrt{x^2+y^2+z^2}$

one QN for e^- spin

So there are 4 quantum numbers for 4 coordinates

Cartesian	Polar	QN
x	r	$n = 1, 2, 3, \ldots$
y	θ	$\ell = 0, 1, 2, \ldots$
z	φ	$m_\ell = -\ell, \ldots, \ell$
ψ_s	ψ_s	$m_s = \pm\frac{1}{2}$

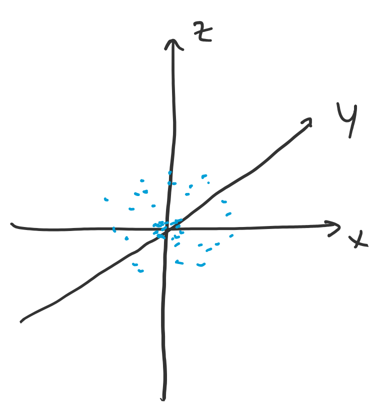


Eigenvalues $= E_n = -\frac{R_y}{n^2} \qquad n = 1, 2, 3, \ldots$

$$R_y = \frac{m_e \cdot e^4}{8\varepsilon_0^2 h^2} = 2.1798723611035 \times 10^{-18}\ \text{J}$$

$$\psi_{n=1,\ell=0,m_\ell=0} \sim e^{-r/a_0} \qquad \frac{4\pi\varepsilon_0\hbar^2}{m_e \cdot e^2} \approx 0.529\ \text{Å}$$

$$P = |\psi|^2 \sim e^{-2r/a_0}$$



1s orbital =